

Multiple spins systems

AX system

Calculate the coupling constant $^1J(^{31}\text{P}^{77}\text{Se})$ to one decimal place, the chemical shift in Hz of phosphorus in molecules containing ^{77}Se , and this shift in ppm, to two decimal places.

- 1(a) $^1J(^{31}\text{P}^{77}\text{Se}) = 760.6 \text{ Hz}$
- 1(b) $\nu_{\text{P}}(^{77}\text{Se}) = 4415.6 \text{ Hz}$
- 1(c) $\nu_{\text{P}}(^{77}\text{Se}) = 36.34 \text{ ppm}$

AB system

Question: The four lines of the ^{31}P AB spectrum shown above are at

18906.223, 18679.424, 18280.477, 18053.680 Hz.

- Calculate ν_{A} , ν_{B} and J_{AB} in Hz, all to one decimal place.

$$J_{\text{AB}} = 226.8 \text{ Hz}; \nu_{\text{M}} = 18479.95 \text{ Hz (152.1 ppm)}; \delta_{\text{AB}} = 583.20 \text{ Hz (4.80 ppm)}$$

$$\nu_{\text{A}} = \nu_{\text{M}} - \delta_{\text{AB}}/2 = 18188.35 \text{ Hz (149.70 ppm)}$$

$$\nu_{\text{B}} = \nu_{\text{M}} + \delta_{\text{AB}}/2 = 18771.55 \text{ Hz (154.50 ppm)}$$

ABX system

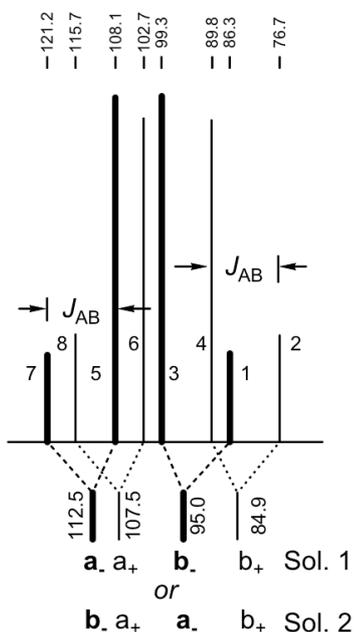
Chemical shift of the AB systems

121.2 ; 115.7 ; 108.1 ; 102.7 ; 99.3 ; 89.8 ; 86.3 ; 76.7

An ABX system will consists in two superimposed AB sub-spectra, with the same J_{AB}

All you need to do is to find the two AB sub-spectra + and -

- Find the value of $^2J_{\text{AB}} = \mathbf{13.1 \text{ Hz}}$
- Solve the AB system for each quartet.



$$c_- = (5+3)/2 = 103.7$$

$$\Delta\nu_{\text{ab}_-} = \delta_- = \sqrt{(7-1)(5-3)} = 17.5$$

$$c_- \pm \delta_-/2 = 103.7 \pm 8.76 = 112.5, 94.9$$

$$c_+ = (6+4)/2 = 96.25$$

$$\Delta\nu_{\text{ab}_+} = \delta_+ = \sqrt{(8-2)(6-4)} = 22.43$$

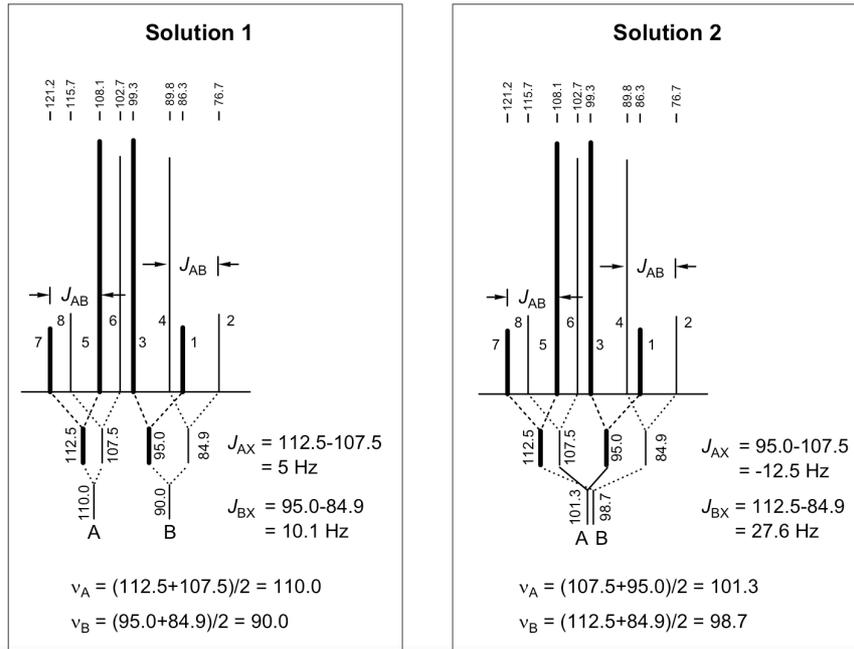
$$c_+ \pm \delta_+/2 = 96.25 \pm 11.21 = 107.5, 85.0$$

At this stage, we know one of the bold lines is **a**, and the other **b**, but we do not know which is which.

Similarly for the thin lines.

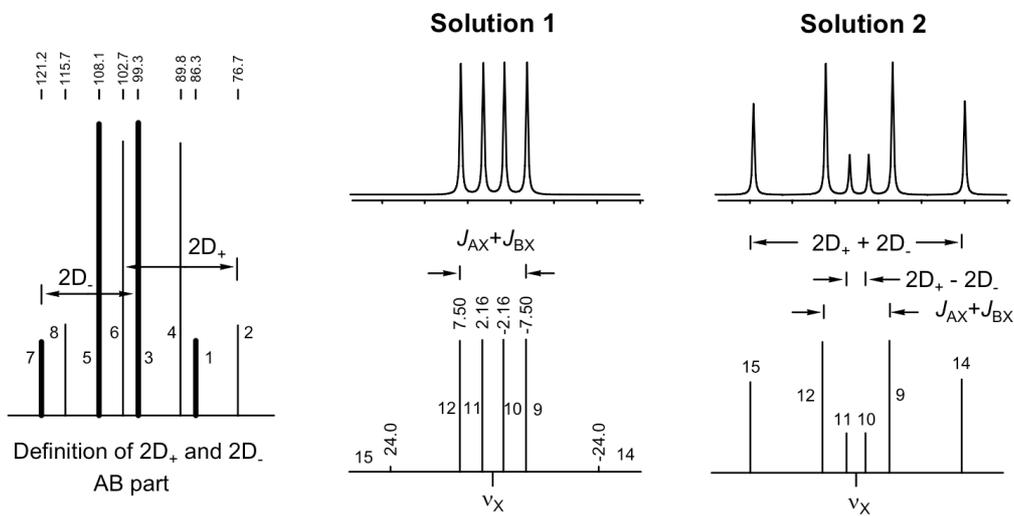
Problem Set - NMR IV - corrections

3. Compare the two solution you have found.



4. Cite some arguments to choose between both. See Lecture slide 45-50
5. The two lines having small intensity are combination lines and decrease in intensity as $\nu_A - \nu_B$ increases. Their intensity is $\sin^2(\phi + \phi_-)$ for an intensity of 1 for the external lines of the X system.

a. ϕ_+ and ϕ_- can be calculated from the formula $\sin^2\phi = J/2C$ (see slide #54) In the solution below $D = C$



b. Give the calculated intensity of the two combinations lines considering the two solutions obtained before and conclude.

Problem Set - NMR IV - corrections

Solution 1

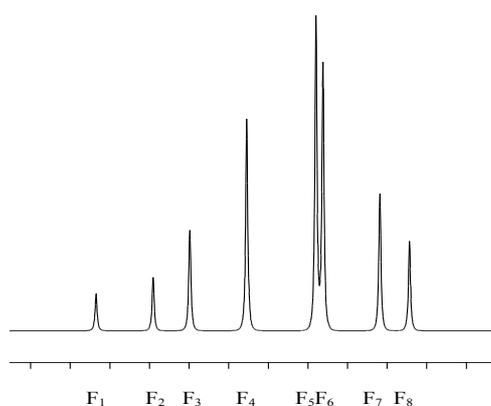
$$\begin{aligned}\Phi_{1+} &= 0.5\arcsin(J_{AB}/2D_+) \\ &= 0.5\arcsin(13.0/26.0) = 15.0 \\ \Phi_{1-} &= 0.5\arcsin(J_{AB}/2D_-) \\ &= 0.5\arcsin(13.0/21.9) = 18.2 \\ i_{10} = i_{11} &= \cos^2(\Phi_{1+} - \Phi_{1-}) \\ &= \cos^2(15.0 - 18.2) = 0.997 \\ i_{14} = i_{15} &= \sin^2(\Phi_{1+} - \Phi_{1-}) \\ &= \sin^2(15.0 - 18.2) = 0.003\end{aligned}$$

Solution 2

$$\begin{aligned}\Phi_{2+} &= \Phi_{1+} \\ &= 15.0 \\ \Phi_{2-} &= 90 - \Phi_{1-} \\ &= 90 - 18.2 = 71.8 \\ i_{10} = i_{11} &= \cos^2(\Phi_{2+} - \Phi_{2-}) \\ &= \cos^2(15.0 - 71.8) = 0.30 \\ i_{14} = i_{15} &= \sin^2(\Phi_{2+} - \Phi_{2-}) \\ &= \sin^2(15.0 - 71.8) = 0.70\end{aligned}$$

$$\arcsin = \sin^{-1}$$

AB₂ system



<i>Frequency</i>	<i>Intensity</i>
218.614	0.368
146.692	0.531
100.000	1.000
28.078	2.106
-59.464	3.101
-68.614	2.632
-140.536	1.363
-178.078	0.894

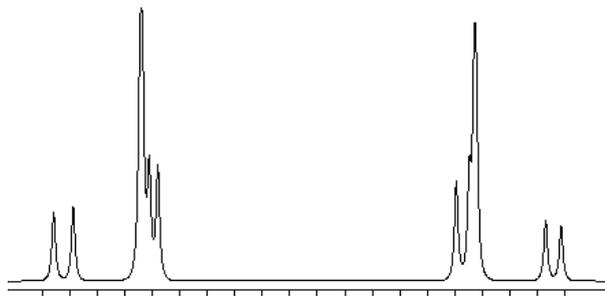
- $\nu_A = 100.00$ Hz
- $\nu_B = 100$ Hz and $J_{AB} = 100$ Hz

AA'XX' system

In strict AA'XX' systems spectra are given by pairs of different species:

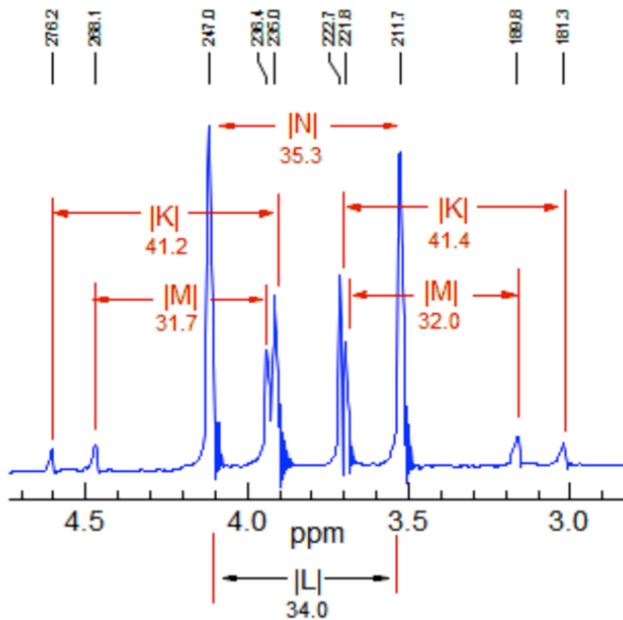
For these systems we define

$ K = J_{AA'} + J_{XX'} $	“ <i>J</i> ” of one ab quartet
$ L = J_{AX} - J_{AX'} $	“ δ ” of both ab quartet
$ M = J_{AA'} - J_{XX'} $	“ <i>J</i> ” of other ab quartet
$ N = J_{AX} + J_{AX'} $	“ <i>J</i> ” of doublet



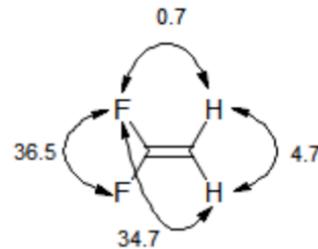
For the molecule 7, the ¹⁹F spectrum shows 10 lines with the following chemical shifts in Hz at:
276.2 ; 268.1 ; 247.0 ; 236.4 ; 235.0 ; 222.7 ; 221.8 ; 211.7 ; 189.8 ; 181.3.

Problem Set - NMR IV - corrections

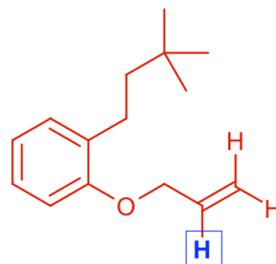
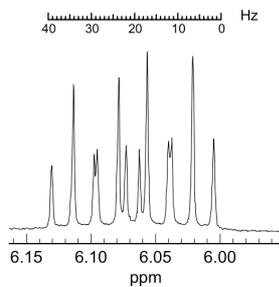


$|K| = |J_{AA'} + J_{XX'}|$ "J" of one ab quartet
 $|L| = |J_{AX} - J_{AX'}|$ "δ" of both ab quartets
 $|M| = |J_{AA'} - J_{XX'}|$ "J" of other ab quartet
 $|N| = |J_{AX} + J_{AX'}|$ "doublet"

If we make the reasonable assumption that $J_{FF} > J_{HH}$ and $J_{HF}(\text{trans}) > J_{HF}(\text{cis})$ we get the following values:

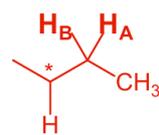
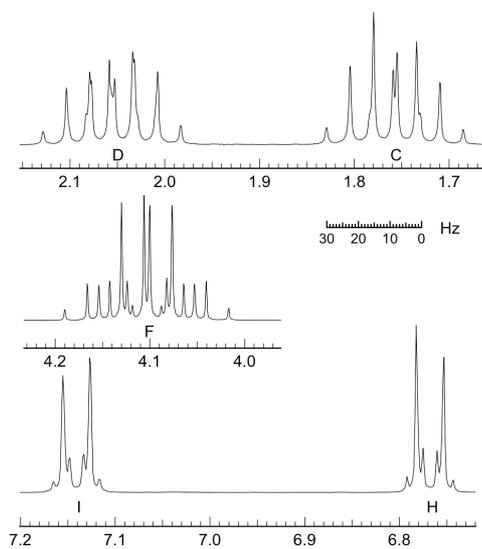


- Determine $N = 35.3$ Hz
- Measure K (41.2 and 41.4 Hz) and M (31.7 Hz and 32.0 Hz).
- Calculate L from the formulae given in equation 7 For K $L = 33.8$ Hz and for $L = 34.2$ Hz.
- Calculate $J_{AA'}$ and $J_{XX'}$ by summing and subtracting K and M .
 $J_{AA'} = (K+M)/2 = 36.5$ Hz; $J_{XX'} = (K-M)/2 = 4.7$ Hz
- Calculate J_{AX} and $J_{AX'}$ by summing and subtracting L and N .
 $J_{AX} = (N+L)/2 = 34.7$ Hz; $J_{AX'} = (N-L)/2 = 0.7$ Hz



ddt, 17, 11, 5

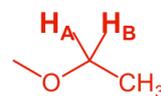
Problem Set - NMR IV - corrections



ABM₃Y

δ 2.05, d quintets, J = 13.4, 7.5

δ 1.75, d quintets, J = 13.5, 7.5



ABX₃

δ 4.13, dq, J = 11, 7

δ 4.08, dq, J = 11, 7

